Numerical Solution of Free-Convection Heat Transfer over a Vertical Cone Embedded in a Non-Newtonian Power-Law Fluid-Saturated Porous Medium with Viscous Dissipation

Mostafa A. A. Mahmoud, Ph.D.[1]

Abstract: A numerical study of the problem of heat transfer to the non-Newtonian power-law fluid over a vertical cone embedded in a porous medium has been investigated, taking into consideration the viscous dissipation effect. The governing equations describing the problem are transformed into a system of nonlinear ordinary differential equations, which is solved numerically using the Chebyshev spectral method. The effects of the power-law index and the viscous dissipation that is characterized by the local Gebhart number on the temperature profiles and the local Nusselt number are discussed. **DOI: 10.1061/(ASCE)EM.1943-7889.0000361.** *© 2012 American Society of Civil Engineers.*

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Introduction

In recent years, the study of thermal convection in porous media has attracted many investigators because of its wide range of applications in geophysical and energy-related engineering problems. A fairly large body of the fundamental research contributions in the area of convective flow in porous media has been reviewed in recent books by Vafai (2005), Pop and Ingham (2001), and Nield and Bejan (2006). The nonlinear characteristics of non-Newtonian fluids in porous media are quite different from those of Newtonian fluids. The prediction of natural convection heat transfer characteristics from heated bodies immersed in a porous medium saturated with a non-Newtonian fluid has a number of thermal engineering applications, such as oil recovery, groundwater pollution, food processing, filtration processes, etc. Non-Newtonian fluids do not satisfy the linear Darcy's law and, therefore, a modified Darcy's law is used to study the flow of these fluids through porous media. Christopher and Middleman (1965) and Dharmadhikari and Kale (1985) have suggested a model for Darcy law applicable to power-law fluids in a porous medium. Since then, the problem has been investigated in different situations by several authors, such as White (1967), Ene and Sanchez-Palencia (1982), Chen and Chen (1987, 1988), Nakayama and Koyama (1991), Nakayama (1993), Mehta and Rao (1994), Bejan (1995), Pascal and Pascal (1997), Yih (1998), and Kumari and Jayanathi (2005).

All of the aforementioned studies neglected the viscous dissipation term in the energy equation. The fundamental role that viscous dissipation played in porous media was first documented by Bejan (1995) and by Nield and Bejan (2006). In the case of a fully developed flow of an incompressible viscous Newtonian fluid in a vertical channel filled by a saturated porous medium, the viscous dissipation term φ, which is appropriate to the Darcy flow, is

$$\varphi = \frac{\mu u^2}{K} \tag{1}$$

where μ = dynamic viscosity of the fluid, and K = permeability. Many researchers, for example, Ene and Sanchez-Palencia (1982), Nakayama and Pop (1989), and Ingham et al. (1990), studied the natural convection in fluid-saturated porous medium taking the viscous dissipation effect [Eq. (1)] into consideration. Nield (2000, 2002) argued that the mathematical form for the viscous dissipation term depends on the model of the drag force in the momentum equation. Nield (2007) pointed out that the equality of the viscous dissipation and the power of the drag, when volume averages are taken, is a fundamental principle that is valid for Darcy flow. This principle leads to a major simplification in the modeling of viscous dissipation. If the Brinkman equation (Nield and Bejan 2006) is used (i.e., if the local drag is correctly modeled by $(\mu/K)u^2 - \mu_{\text{eff}} u(\partial^2 u/\partial y^2)$), then the viscous dissipation term is

$$\varphi = \frac{\mu}{K} u^2 - \mu_{\text{eff}} u \frac{\partial^2 u}{\partial y^2} \tag{2}$$

where μ_{eff} = effective viscosity in the Brinkman term. Eq. (2) is valid for finite values of the Darcy number D_a.

Al-Hadhrami et al. (2002, 2003) proposed the following model for the viscous dissipation term, which is valid at infinite number D_a:

$$\varphi = \frac{\mu u^2}{K} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \tag{3}$$

where $D_a = K/L^2$ and L = characteristic length scale. So, Eq. (2), which is based on the Brinkman drag, is consistent with the power of the drag principle but is invalid for infinite D_a, and Eq. (3) is valid at infinite D_a, consistent with the power of the drag principle at infinite D_a, but is not consistent with that principle at finite D_a if

[1]Associate Professor, Dept. of Mathematics, Faculty of Science, Benha Univ., Benha 13518, Egypt. E-mail: mostafabdelhameed@yahoo.com

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